

## Monday 19 October 2020 – Afternoon

### A Level Mathematics B (MEI)

#### H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours



**You must have:**

- the Printed Answer Booklet
- the Insert
- a scientific or graphical calculator

#### INSTRUCTIONS

- Use black ink. You can use an HB pencil, but only for graphs and diagrams.
- Write your answer to each question in the space provided in the **Printed Answer Booklet**. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.
- Fill in the boxes on the front of the Printed Answer Booklet.
- Answer **all** the questions.
- Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.
- Give your final answers to a degree of accuracy that is appropriate to the context.
- Do **not** send this Question Paper for marking. Keep it in the centre or recycle it.

#### INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [ ].
- This document has **12** pages.

#### ADVICE

- Read each question carefully before you start your answer.

## Formulae A Level Mathematics B (MEI) (H640)

**Arithmetic series**

$$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a + (n-1)d\}$$

**Geometric series**

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

**Binomial series**

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n \quad (n \in \mathbb{N}),$$

$$\text{where } {}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots \quad (|x| < 1, n \in \mathbb{R})$$

**Differentiation**

$f(x)$	$f'(x)$
$\tan kx$	$k \sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

$$\text{Quotient Rule } y = \frac{u}{v}, \quad \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

**Differentiation from first principles**

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Integration**

$$\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + c$$

$$\int f'(x)(f(x))^n dx = \frac{1}{n+1}(f(x))^{n+1} + c$$

$$\text{Integration by parts } \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

**Small angle approximations**

$$\sin \theta \approx \theta, \quad \cos \theta \approx 1 - \frac{1}{2}\theta^2, \quad \tan \theta \approx \theta \quad \text{where } \theta \text{ is measured in radians}$$

### Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \quad \left( A \pm B \neq \left(k + \frac{1}{2}\right)\pi \right)$$

### Numerical methods

$$\text{Trapezium rule: } \int_a^b y \, dx \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \dots + y_{n-1})\}, \text{ where } h = \frac{b-a}{n}$$

$$\text{The Newton-Raphson iteration for solving } f(x) = 0: x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

### Probability

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B) \quad \text{or} \quad P(A|B) = \frac{P(A \cap B)}{P(B)}$$

### Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

$$\text{Standard deviation, } s = \sqrt{\text{variance}}$$

### The binomial distribution

$$\text{If } X \sim B(n, p) \text{ then } P(X = r) = {}^n C_r p^r q^{n-r} \text{ where } q = 1 - p$$

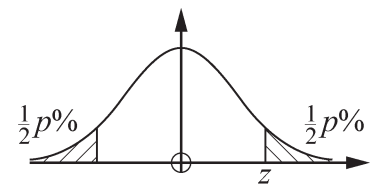
Mean of  $X$  is  $np$

### Hypothesis testing for the mean of a Normal distribution

$$\text{If } X \sim N(\mu, \sigma^2) \text{ then } \bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right) \text{ and } \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$$

### Percentage points of the Normal distribution

$p$	10	5	2	1
$z$	1.645	1.960	2.326	2.576



### Kinematics

Motion in a straight line

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

$$v^2 = u^2 + 2as$$

$$s = vt - \frac{1}{2}at^2$$

Motion in two dimensions

$$\mathbf{v} = \mathbf{u} + \mathbf{a}t$$

$$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$$

$$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$$

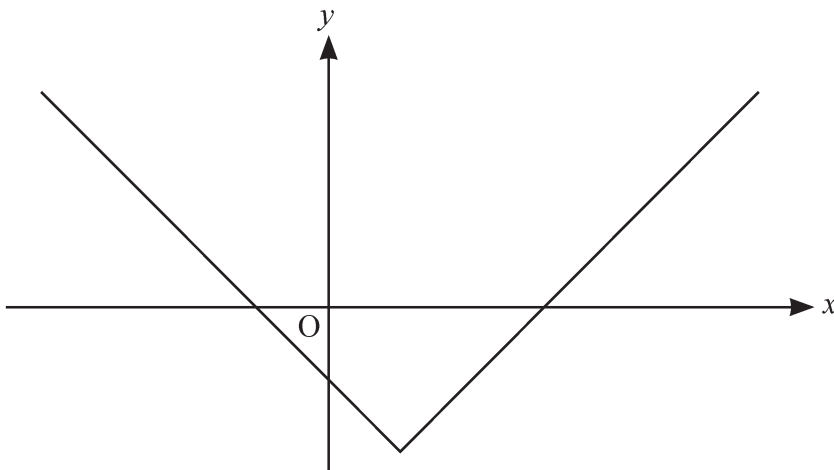
$$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$$

Answer **all** the questions.

**Section A** (60 marks)

1 Find the value of  $\sum_{r=1}^5 2^r(r-1)$ . [2]

2 The graph of  $y = |1-x| - 2$  is shown in Fig. 2.



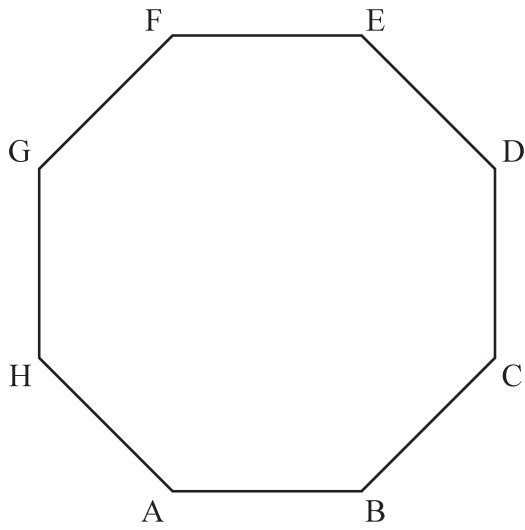
**Fig. 2**

Determine the set of values of  $x$  for which  $|1-x| > 2$ . [4]

3 A particular phone battery will last 10 hours when it is first used. Every time it is recharged, it will only last 98% of its previous time.

Find the maximum total length of use for the battery. [3]

- 4 Fig. 4 shows the regular octagon ABCDEFGH.



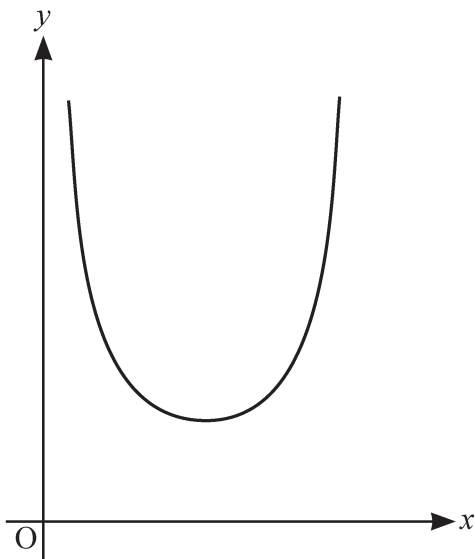
**Fig. 4**

$\overrightarrow{AB} = \mathbf{i}$ ,  $\overrightarrow{CD} = \mathbf{j}$ , where  $\mathbf{i}$  is a unit vector parallel to the  $x$ -axis and  $\mathbf{j}$  is a unit vector parallel to the  $y$ -axis.

Find an exact expression for  $\overrightarrow{BC}$  in terms of  $\mathbf{i}$  and  $\mathbf{j}$ .

[3]

- 5 Fig. 5 shows part of the curve  $y = \operatorname{cosec} x$  together with the  $x$ - and  $y$ -axes.



**Fig. 5**

- (a) For the section of the curve which is shown in Fig. 5, write down
- (i) the equations of the two vertical asymptotes, [2]
  - (ii) the coordinates of the minimum point. [1]
- (b) Show that the equation  $x = \operatorname{cosec} x$  has a root which lies between  $x = 1$  and  $x = 2$ . [2]
- (c) Use the iteration  $x_{n+1} = \operatorname{cosec}(x_n)$ , with  $x_0 = 1$ , to find
- (i) the values of  $x_1$  and  $x_2$ , correct to 5 decimal places, [1]
  - (ii) this root of the equation, correct to 3 decimal places. [1]
- (d) There is another root of  $x = \operatorname{cosec} x$  which lies between  $x = 2$  and  $x = 3$ .
- Determine whether the iteration  $x_{n+1} = \operatorname{cosec}(x_n)$  with  $x_0 = 2.5$  converges to this root. [1]
- (e) Sketch the staircase or cobweb diagram for the iteration, starting with  $x_0 = 2.5$ , on the diagram in the Printed Answer Booklet. [3]

- 6 (a) (i) Write down the derivative of  $e^{kx}$ , where  $k$  is a constant.
- (ii) A business has been running since 2009. They sell maths revision reso
- Give a reason why an exponential growth model might be suitable fo  
for the business. [1]

Fig. 6 shows the relationship between the annual profits of the business in thousands of pounds ( $y$ ) and the time in years after 2009 ( $x$ ). The graph of  $\ln y$  plotted against  $x$  is approximately a straight line.

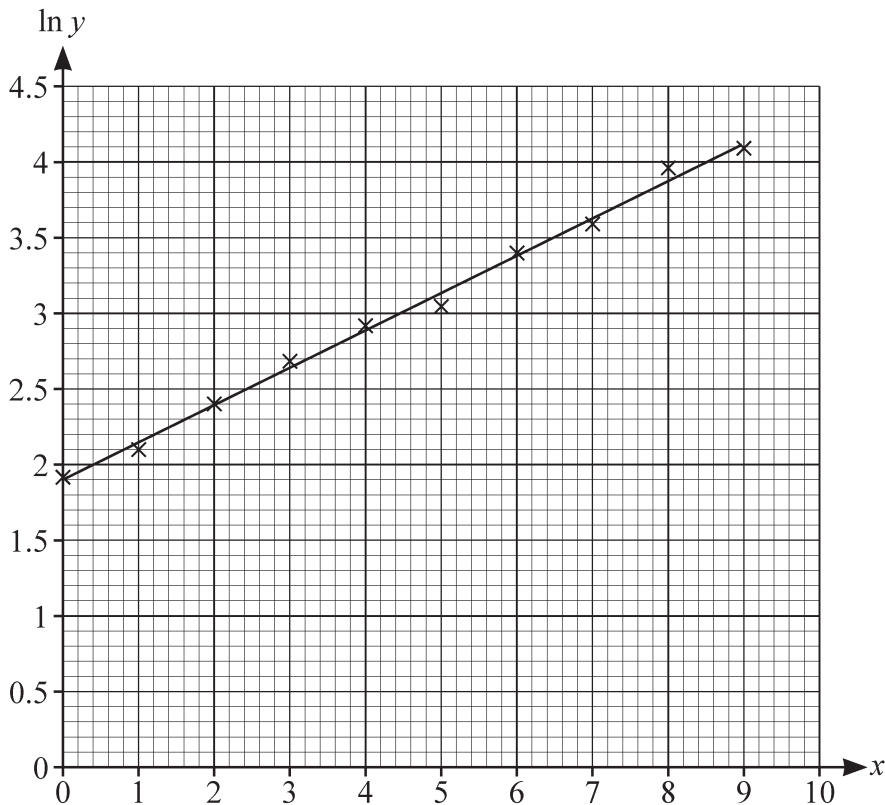


Fig. 6

- (b) Show that the straight line is consistent with a model of the form  $y = Ae^{kx}$ , where  $A$  and  $k$  are constants. [2]
- (c) Estimate the values of  $A$  and  $k$ . [4]
- (d) Use the model to predict the profit in the year 2020. [3]
- (e) How reliable do you expect the prediction in part (d) to be? Justify your answer. [1]

- 7 (a) Express  $\frac{1}{x} + \frac{1}{A-x}$  as a single fraction.

The population of fish in a lake is modelled by the differential equation

$$\frac{dx}{dt} = \frac{x(400-x)}{400}$$

where  $x$  is the number of fish and  $t$  is the time in years.

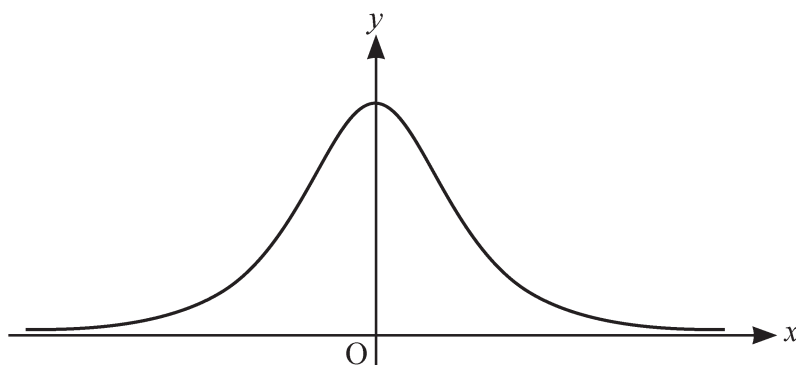
When  $t = 0$ ,  $x = 100$ .

- (b) **In this question you must show detailed reasoning.**

Find the number of fish in the lake when  $t = 10$ , as predicted by the model.

[8]

- 8 (a) The curve  $y = \frac{1}{(1+x^2)^2}$  is shown in Fig. 8.



**Fig. 8**

- (i) Show that  $\frac{d^2y}{dx^2} = \frac{20x^2 - 4}{(1+x^2)^4}$ .

[5]

- (ii) **In this question you must show detailed reasoning.**

Find the set of values of  $x$  for which the curve is concave downwards.

[3]

- (b) Use the substitution  $x = \tan \theta$  to find the exact value of  $\int_{-1}^1 \frac{1}{(1+x^2)^2} dx$ .

[8]



Answer **all** the questions.

**Section B** (15 marks)

The questions in this section refer to the article on the Insert. You should read attempting the questions.

9 (a) Show that if  $a = 1$  and  $b > 1$  then  $a^b < b^a$ . [2]

(b) Find integer values of  $a$  and  $b$  with  $b > a > 1$  and  $a^b$  not greater than  $b^a$  (a counter example to the conjecture given in lines 7–8). [1]

10 In this question you must show detailed reasoning.

Show that  $\int_e^\pi \frac{1}{x} dx = \ln \pi - 1$  as given in line 37. [2]

11 Show that  $e^x$  is an increasing function for all values of  $x$ , as stated in line 39. [2]

12 (a) Show that the only stationary point on the curve  $y = \frac{\ln x}{x}$  occurs where  $x = e$ , as given in line 45. [3]

(b) Show that the stationary point is a maximum. [3]

(c) It follows from part (b) that, for any positive number  $a$  with  $a \neq e$ ,

$$\frac{\ln e}{e} > \frac{\ln a}{a}.$$

Use this fact to show that  $e^a > a^e$ . [2]

**END OF QUESTION PAPER**



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